

A Level Pure Paper Mark Scheme: MathsFlex Tutoring

1.

$g(4) = \frac{3(4) + 4}{4 - 2} = 8 \Rightarrow gg(4) = \frac{3(8) + 4}{8 - 2}$	M1
$gg(4) = \frac{14}{3}$ oe	A1
Range : $3 < y \leq 8$	B1
$y = \frac{3x + 4}{x - 2} \Rightarrow yx - 2y = 3x + 4 \Rightarrow yx - 3x = 2y + 4$	M1
$x(y - 3) = 2y + 4 \Rightarrow x = \frac{2y + 4}{y - 3}$	M1
$g^{-1}(x) = \frac{2x + 4}{x - 3}, 3 < x \leq 8$	A1ft

2.

2	Attempts $f(-2) = 3 \times (-2)^3 - 2a \times (-2)^2 - 28 \times (-2) - 8a = 0$	M1
	Solves linear equation $16a = 32$	M1
	$a = 2$ cao	A1

3.

3a	Deduces that the gradient of PA is $-\frac{1}{3}$	M1
	Finding the equation of a line with gradient $-\frac{1}{3}$ and point (6, 4) $y - 4 = -\frac{1}{3}(x - 6)$	M1
	Completes proof $3y + x = 18$	A1
3b	Solves $3y + x = 18$ and $y = 3x + 1$ simultaneously	M1
	$P = (1.5, 5.5)$	A1
	Length $PA = \sqrt{(6 - 1.5)^2 + (4 - 5.5)^2} (= \sqrt{22.5})$	M1
	Equation of C is $(x - 6)^2 + (y - 4)^2 = 22.5$	A1
3c	Attempts to find where $y = 3x + k$ meets C using $\overrightarrow{OA} + \overrightarrow{PA}$	M1
	Substitutes their (10.5, 2.5) in $y = 3x + k$ to find k	M1
	$k = -29$	A1

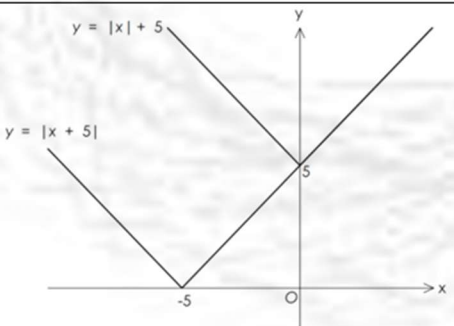
4.

$\{LHS = \frac{\cos 6\theta \cos 2\theta + \sin 6\theta \sin 2\theta}{\sin 2\theta \cos 2\theta}$	M1
$= \frac{\cos(6\theta - 2\theta)}{\sin 2\theta \cos 2\theta} = \frac{\cos 4\theta}{\sin 2\theta \cos 2\theta}$	A1
$\sin 2\theta \cos 2\theta = \frac{1}{2} \sin 4\theta$	dM1
$\frac{\cos 4\theta}{\frac{1}{2} \sin 4\theta} = 2 \cot 4\theta$	A1
$\left\{ \frac{\cos 6\theta}{\sin 2\theta} + \frac{\sin 6\theta}{\cos 2\theta} = 5 \Rightarrow \right\} 2 \cot 4\theta = 5 \Rightarrow 2 \left(\frac{1}{\tan 4\theta} \right) = 5$	M1
Rearrange to give $\tan 4\theta = k$; $k \neq 0$ and applies $\arctan k$ $\left\{ 90^\circ < \theta < 180^\circ, \tan 4\theta = \frac{2}{5} \Rightarrow \right\}$	dM1
$\theta = \text{awrt } 94.5^\circ, 140.5^\circ$	A1

5.

a	Total time for 8 km = 48 minutes + $8 \times 1.03 + 8 \times 1.03^2$	M1
	= 64.7272 minutes = 64 minutes 44 seconds	A1
b	7 th km is $8 \times 1.03 = 8 \times 1.03^1$ 8 th km is $8 \times 1.03 \times 1.03 = 8 \times 1.03^2$ 9 th km is $8 \times 1.03 \times 1.03 \times 1.03 = 8 \times 1.03^3$ Hence the time for the r th km is $8 \times 1.03^{r-6}$	B1
c	Attempts the total time for the race Eg 48 minutes + $\sum_{r=7}^{30} 8 \times 1.03^{r-6}$ minutes	M1
	Uses the series formula to find an allowable sum Eg Time for 7 th to 30 th km = $\frac{8.24(1.03^{24} - 1)}{1.03 - 1} = (283.67...)$	M1
	Correct calculation that leads to the total time Eg Total time = $48 + \frac{8.24(1.03^{24} - 1)}{1.03 - 1}$	A1
	Total time = awrt 331 minutes 40 seconds	A1

6.

a	Eg $m = \sqrt{8}, n = \sqrt{2}$	M1
	$\frac{m}{n} = \frac{\sqrt{8}}{\sqrt{2}} = 2$ $\Rightarrow$ statement untrue or 2 is not irrational or 2 is rational	A1
b		B1
		M1
	The graph of $y = x + 5$ is either the same or above the graph of $y = x + 5 $ {for corresponding values of x } or when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = x + 5$ is above the graph of $y = x + 5 $	A1

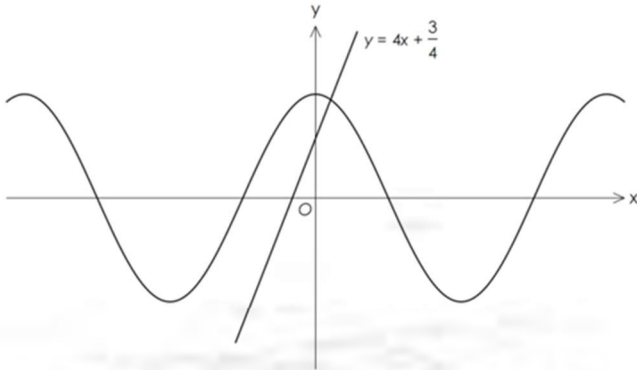
7.

a	$\sqrt{\frac{1+10x}{1-x}} = (1+10x)^{\frac{1}{2}} \times (1-x)^{-\frac{1}{2}}$	B1
	$(1+10x)^{\frac{1}{2}} = 1 + \frac{1}{2} \times (10x) + \frac{\frac{1}{2} \times (-\frac{1}{2})}{2} \times (10x)^2$	M1
	$(1-x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right) \times (-x) + \frac{\left(-\frac{1}{2}\right) \times \left(-\frac{3}{2}\right)}{2} \times (-x)^2$	M1
	$(1+10x)^{\frac{1}{2}} = 1 + 5x - \frac{25}{2}x^2$ and $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{3}{8}x^2$	A1
	$(1+10x)^{\frac{1}{2}} \times (1-x)^{-\frac{1}{2}} = \left(1 + 5x - \frac{25}{2}x^2\right) \times \left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + 5x + \frac{5}{2}x^2 - \frac{25}{2}x^2$ $= A + Bx + Cx^2$	dM1
	$= 1 + \frac{11}{2}x - \frac{77}{8}x^2$	A1
b	Expression is valid $ x < \frac{1}{10}$ Should not use $x = \frac{1}{2}$ as $\frac{1}{2} > \frac{1}{10}$	B1
c	Substitutes $x = \frac{1}{34}$ into $\sqrt{\frac{1+10x}{1-x}} \approx 1 + \frac{11}{2}x - \frac{77}{8}x^2$	M1
	$\sqrt{\frac{4}{3}} = \frac{10667}{9248}$	A1
	So $\sqrt{12} \approx \frac{32001}{9248}$	A1

8.

a	$x^2 \tan y = 6 \Rightarrow 2x \tan y + x^2 \sec^2 y \frac{dy}{dx} = 0$	M1 A1
	Full method to get $\frac{dy}{dx}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + \frac{36}{x^4}$	M1
	$\frac{dy}{dx} = \frac{-2x \times \frac{6}{x^2}}{x^2 \left(1 + \frac{36}{x^4}\right)} = \frac{-12x}{x^4 + 36}$	A1
b	$\frac{dy}{dx} = \frac{-12x}{x^4 + 36}$ $\Rightarrow \frac{d^2y}{dx^2} = \frac{-12 \times (x^4 + 36) - (-12x) \times 4x^3}{(x^4 + 36)^2} = \frac{36(x^4 - 12)}{(x^4 + 36)^2}$ oe	M1 A1
	States that when $x < \sqrt[4]{12} \Rightarrow \frac{d^2y}{dx^2} < 0$ when $x = \sqrt[4]{12} \Rightarrow \frac{d^2y}{dx^2} = 0$ AND when $x > \sqrt[4]{12} \Rightarrow \frac{d^2y}{dx^2} > 0$ giving a point of inflection when $x = \sqrt[4]{12}$	A1

9.

ia		B1
	For an allowable linear graph and explaining that there is only one intersection	B1
ib	$\cos x - 4x - \frac{3}{4} = 0 \Rightarrow 1 - \frac{x^2}{2} - 4x - \frac{3}{4} = 0$	M1
	Solves their $2x^2 + 16x - 1 = 0$	dM1
	Allow awrt 0.062 but accept $\frac{-8 + \sqrt{66}}{2}$	A1

10.

Chooses a suitable method for $\int_0^4 3x\sqrt{x+5}dx$		M1
Award for - Using a valid substitution $u = \dots$, changing the terms to u's - Integrating and using appropriate limits.		
Substitution $u = \sqrt{x+5} \Rightarrow \frac{dx}{du} = 2u$ oe	Substitution $u = x+5 \Rightarrow \frac{dx}{du} = 1$ oe	B1
$\int 3x\sqrt{x+5}dx$ $= \int A(u^2 \pm 5)u^2 du$	$\int 3x\sqrt{x+5}dx$ $= \int A(u \pm 5)\sqrt{u} du$	M1
$= Pu^5 \pm Qu^3$	$= Su^{\frac{5}{2}} \pm Tu^{\frac{3}{2}}$	dM1
$= \frac{6}{5}u^5 - \frac{30}{3}u^3$	$= \frac{6}{5}u^{\frac{5}{2}} - \frac{30}{3}u^{\frac{3}{2}}$	A1
Uses limits 3 and $\sqrt{5}$ the correct way around	Uses limits 9 and 5 the correct way around	ddM1
$= \frac{108}{5} + \frac{100\sqrt{5}}{5}$ $= \frac{4}{5}(27 + 25\sqrt{5})$		A1

11.

a	i. $y \times \frac{dx}{dt} = 3\sin 2t \times 4\cos t$ or $3 \times 2\sin t \cos t \times 4\cos t$	M1
	(Area =) $\int 3\sin 2t \times 4\cos t dt = \int 3 \times 2\sin t \cos t \times 4\cos t dt$ or $\int 3\sin 2t \times 4\cos t dt = \int 24\sin t \cos^2 t dt$	dM1
	(Area =) $\int_0^{\frac{\pi}{2}} 24\sin t \cos^2 t dt$	A1
	ii. $\int 24\sin t \cos^2 t dt = -8\cos^3 t$	M1 A1
	Area = $[-8\cos^3 t]_0^{\frac{\pi}{2}} = 0 - (-8) = 8$	A1
b	$3\sin 2t = 2.7 \Rightarrow \sin 2t = \frac{2.7}{3}$	M1
	$t = 0.55988\dots, 1.01091\dots$	A1
	Attempts to find the x values at both t values	dM1
	$t = 0.55988\dots \Rightarrow x = 2.12435\dots$ $t = 1.01091\dots \Rightarrow x = 3.38926\dots$	A1
	Width of path = 1.3 metres	A1

12.

a	$\frac{dr}{dt} \propto \pm \frac{1}{r^2}$ or $\frac{dr}{dt} = \pm \frac{k}{r^2}$ (for k or a numerical k)	M1
	$\int r^2 dr = \int \pm k dt \Rightarrow \dots$ (for k or a numerical k)	M1
	$\frac{1}{3}r^3 = \pm kt \{+ c\}$	A1
	$t = 0, r = 6$ and $t = 7, r = 2$ gives $\frac{1}{3}r^3 = -\frac{208}{21}t + 72$, where r, in mm, is the radius {of the mint} and t, in minutes, is the time from when it {the mint} was placed in the mouth $t = 0, r = 6$ and $t = 420, r = 2$ gives $\frac{1}{3}r^3 = -\frac{52}{315}t + 72$, where r, in mm, is the radius {of the mint} and t, in seconds, is the time from when it {the mint} was placed in the mouth	M1 A1
b	$r = 0 \Rightarrow 0 = -\frac{208}{21}t + 72 \Rightarrow 0 = -208t + 1512 \Rightarrow t = \dots$	M1
	time = 7 minutes 16 seconds	A1
c	Suggests a suitable limitation of the model. Eg - Model does not consider how the mint is sucked - Model does not consider whether the mint is bitten - Model is limited for times up to 7 minutes 16 seconds oe - Not valid for times greater than 7 minutes 16 seconds oe - Mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked - The model indicates that the radius of the mint is negative after it dissolves - Model does not consider the temperature of the mouth - Model does not consider rate of saliva production - Mint could be swallowed before it dissolves in the mouth	B1

13.

a	$\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} \Rightarrow \overrightarrow{CM} = -2\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1
	$\Rightarrow \overrightarrow{CM} = -\frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$	A1
b	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda \overrightarrow{CM}$	M1
	$\overrightarrow{ON} = 3\mathbf{a} + \lambda\left(-\frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON} = \left(3 - \frac{5}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$	A1
c	$\left(3 - \frac{5}{2}\lambda\right) = 0 \Rightarrow \lambda = \dots$	M1
	$\lambda = \frac{6}{5} \Rightarrow \overrightarrow{ON} = \frac{3}{5}\mathbf{b} \Rightarrow \left\{\overrightarrow{NB} = \frac{2}{5}\mathbf{b}\right\} \Rightarrow ON : NB = 3 : 2$	A1ft

14.

1. Find Inverses	B1	Correct $f^{-1}(x) = \sqrt{x}$ (oe)
	M1	Attempt to find $g^{-1}(x)$ (e.g., $x = -4y + 4$)
	A1	Correct $g^{-1}(x) = \frac{4-x}{4}$ or $1 - \frac{x}{4}$ (oe)
2. Substitute Values	B1	Correct evaluation of $h(g(2)) = 3^{g(2)} = 3^{-4} = \frac{1}{81}$
	M1	Substitute all components into the equation: $\frac{1}{81} + \sqrt{3^x} = 12(\frac{4-3^x}{4})$
3. Quadratic Form	M1	Simplify and use substitution (e.g., let $u = \sqrt{3^x}$ or $3^{x/2}$) to get a quadratic: $3u^2 + u - \frac{971}{81} = 0$ (oe)
4. Solve for x	M1	Solve quadratic for u (using formula or calculator) and set $3^{x/2} = u$
	A1 CSO	Final answer $x \approx 1.11$ (or exact equivalent $\log_3(u^2)$). Must have no errors leading to this point.
Total Marks	8	